

| Example 1     |  |
|---------------|--|
|               | $\{1,2\}\succ_1\{1\}\succ_1\{1,2,3\}\succ_1\{1,3\}$  |
|               | $\{1,2\}\succ_2\{2\}\succ_2\{1,2,3\}\succ_2\{2,3\}$  |
|               | $\{1,2,3\} \succ_3 \{2,3\} \succ_3 \{1,3\} \succ_3 \{3\}$  |
|               | in the core and is individually stable.<br>Nash stable partitions.   |
| {{1},{2},{3}} | $\{1,2\}$ is preferred by both agent 1 and 2, hence not NS, not IS.  |
| {{1,2},{3}}   | {1,2,3} is preferred by agent 3, so it is not NS, as agents<br>1 and 3 are worse off, it is not a possible move for IS.<br>no other move is possible for IS. |
| {{1,3},{2}}   | agent 1 prefers to be on its own (not NS, then, not IS).   |
| {{2,3},{1}}   | agent 2 prefers to join agent 1,<br>and agent 1 is better off, hence not NS, not IS.   |
| {{1,2,3}}     | agents 1 and 2 have an incentive to form a singleton,<br>hence not NS, not IS.   |

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## Example 3

 $\{1,2\} \succ_1 \{1,3\} \succ_1 \{1\} \succ_1 \{1,2,3\}$  $\{2,3\} \succ_2 \{1,2\} \succ_2 \{2\} \succ_2 \{1,2,3\}$  $\{1,3\} \succ_2 \{2,3\} \succ_2 \{3\} \succ_2 \{1,2,3\}$ 

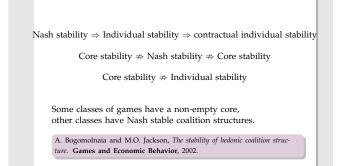
$$\{1,5\} \succ_3 \{2,5\} \succ_3 \{3\} \succ_3 \{1,2,5\}$$
  
The core is empty (similar argument as for example 2).

There is no Nash stable partition or individually stable partition. But there are three contractually individually stable CSs:  $\{\{1,2\},\{3\}\},\{\{1,3\},\{2\}\},\{\{2,3\},\{1\}\}\}$ .

For {{1,2},{3}}:

- {{1},(2,3}}: agents 2 and 3 benefit, hence {{1,2},{3}} is not Nash or individually stable. however, agent 1 is worse off, hence not a possible move for CIS.
- {{2},{1,3}}: agent 1 has no incentive to join agent 3.
- {[1], {2], {3}}: neither agent 1 or 2 has any incentive to form a
- singleton coalition.

$$\begin{split} & \{1,2\}\succ_1\{1,3\}\succ_1\{1,2,3\}\succ_1\{1\}\\ & \{2,3\}\succ_2\{1,2\}\succ_2\{1,2,3\}\succ_2\{2\}\\ & \{1,3\}\succ_3\{2,3\}\succ_3\{1,2,3\}\succ_3\{3\} \end{split}$$
 The core is empty.  $\begin{cases} \{1,2\},\{3\}, \{1,2\}, \{1,3\}, \{2,3\} \text{ and } \{1,2,3\} \text{ are blocking }\\ & \{\{1,2\},\{3\}, \{2,3\}\} \text{ is blocking }\\ & \{\{1,2\},\{3\}, \{1,2\}\} \text{ is blocking }\\ & \{\{1,2,3\}\}, \{1,2\} \text{ is blocking }\\ & \{\{1,2,3\}\}, \{1,2\} \text{ is blocking }\\ & \{\{1,2,3\}\}, \{1,3\} \text{ is blocking }\\ & \{\{1,2,3\}\}, \{1,2\} \text{ is blocking }\\ & \{\{1,2,3\}\}, \{1,3\} \text{ is blocking }\\ & \{\{1,2,3\}\}\} \text{ is the unique Nash stable partition, unique individually stable partition (no agent has any incentive to leave the grand coalition). \end{split}$ 



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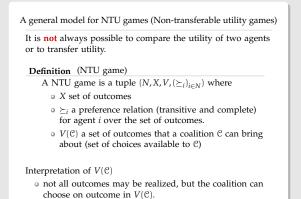
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## Representations

- Anonymous preferences player's preferences depend only on the size of the coalition.
- **Individually rational coalition lists (IRCLs)** list only those coalitions that are preferred to singletons.
- Additively separable game (ASG). A game *G* is AS if there exists an  $|N| \times |N|$  matrix *M* such that  $\mathcal{C}_1 \succeq_i \mathcal{C}_2$  iff  $\sum_{j \in \mathcal{C}_1} M(i,j) \ge \sum_{k \in \mathcal{C}_2} M(i,k)$ .
- **Friends and Enemies:** each player has a set of friends *F* and a set of enemies *E* and use the number of members that are friends or enemies to evaluate their preferences (friends appreciation: which coalition has most friends, break ties with number of enemies, and enemies aversion: do the opposite)
- B and W-preferences: use a ranking over individuals and base the preferences on the best and worst member of the coalition.
- Hedonic Coalition Nets based on MC-nets.

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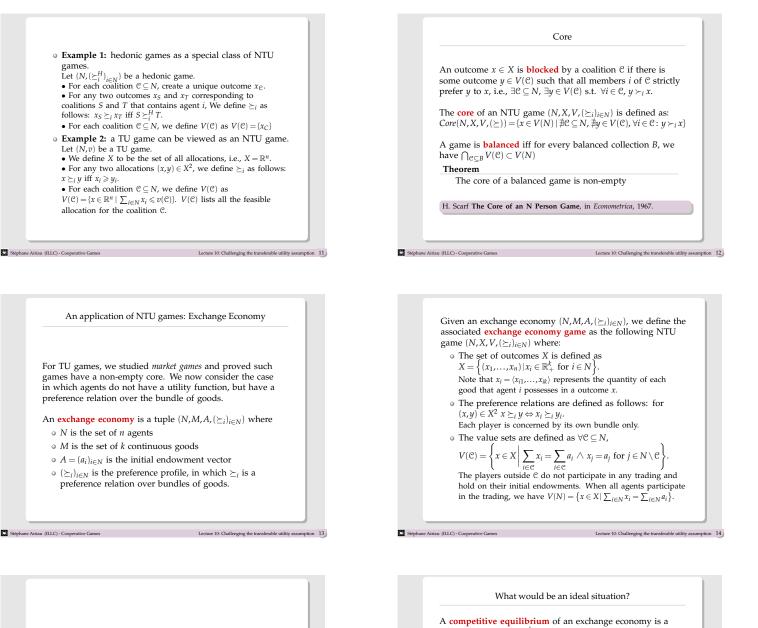
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• Effectivity function: set of choices that can be enforced by coalition C.

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Let us assume we can define a price  $p_r$  for a unit of good r. The idea would be to exchange the goods at a **constant price** during the negotiation.

Let us define a price vector  $p \in \mathbb{R}_+^k$ . The amount of each good that agent *i* possesses is  $x_i \in \mathbb{R}_+^k$ .

The total cost of agent *i*'s bundle is  $p \cdot x_i = \sum p_r x_{i,r}$ .

Since the initial endowment of agent *i* is  $a_{i}$ , the agent has at his disposal an amount  $p \cdot a_i$ , and *i* can afford to obtain a bundle  $y_i$  such that  $p \cdot y_i \leq p \cdot a_i$ .

Theorem

**pair** (p, x) where  $p \in \mathbb{R}^k_+$  is a price vector and  $x \in \{(x_1, \dots, x_n) | x_i \in \mathbb{R}^k_+ \text{ for } i \in N\}$  such that

•  $\forall i \in N \ \forall y_i \in \mathbb{R}^k_+ \ (p \cdot y_i \leq p \cdot a_i) \Rightarrow x_i \succeq_i y_i$ 

then a competitive equilibrium exists.

one of its most favorites outcomes

it possesses the best outcome.

•  $\sum_{i \in N} x_i = \sum_{i \in N} a_i$  (the allocation results from trading) •  $\forall i \in N, \ p \cdot x_i \leq p \cdot a_i$  (each agent can afford its allocation)

Among all the allocations that an agent can afford, it obtains

Using the price vector and the allocation, each agent believes

Let  $(N, M, A, (\succeq_i)_{i \in N})$  be an exchange economy. If each preference relation  $\succeq_i$  is continuous and strictly convex,

